Ibn al-Haytham's Scientific Research Programme

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2.1 Introduction

For the vast majority of historians, and, more generally, of laymen, Ibn al-Haytham's major contribution concerns the vision in all its aspects (physical, physiological and psychological) and, namely, the causes of perceptual and cognitive effects. The reform of Ibn al-Haytham, according to them, was mainly to abandon the traditional theory of vision, to a new one. Henceforth he belongs to ancient and mediaeval traditions, in spite of this reform, in so far that he was concerned with vision and sight.

I will argue here that this reform was a minor consequence of a more general and more fundamental research programme, and even his conception of the science of optics is quite different as so far that his main task was about light, its fundamental properties and how they determine its physical behaviour, as reflection, refraction, focalization, etc.

Some historians of optics consider that, up to the seventeenth century in Europe, the science in optics before Kepler was aimed primarily at explaining vision. The merest glance at the optical works of Ibn al-Haytham leaves no doubt that this global judgement is far from being correct. Indeed, this statement is correct as far as it concerns the history of optics before the shift done by Ibn al-Haytham and the reform he accomplished. Successor of Ptolemy, al-Kindī and Ibn Sahl, to mention only a few, he unified the different branches of optics: optics, dioptrics, anaclastics, meteorological optics, etc. This unification was possible only for a mathematician who focused on light, and not on vision. Nobody, as far as I know, before Ibn al-Haytham, wrote such books titled: *On Light*; *On the Light of the Moon*; *On the Light of the Stars*; *On the Shadows*, among others, in which nothing concerns sight. At the same time, three books from his famous *Book of Optics* are devoted strictly to the theory of light. None of the authors before him, who were mainly interested in vision, wrote a very important contribution on physical optics such as the one on *The Burning Sphere*.

I begin by quoting the expression which Ibn al-Haytham repeated more than once in his different writings on optics. At the beginning of this famous *Book of Optics*, he writes:

> Our subject is obscure and the way leading to knowledge of its nature difficult, moreover our inquiry requires a combination of the natural and mathematical sciences.[1]

But such a combination in optics, for instance, requires one to examine the entire foundations and to invent the means and the procedures to apply mathematics on the ideas of natural phenomena. For Ibn al-Haytham, it was the only way to obtain a rigorous body of knowledge.

Why this particular turn, at that time? Let me remind that Ibn al-Haytham lived in the turn of the first millennium. He was the heir of two centuries of scientific research and scientific translations, in mathematics, in astronomy, in statics, in optics, etc. His time was of intense research in all these fields. He himself wrote more in mathematics and in astronomy than in optics per se. According to early bio-bibliographers, Ibn al-Haytham wrote 25 astronomical works: twice as many works on the subject as he did in optics. The number of his writings alone indicates the huge size of the task accomplished by him and the importance of astronomy in his life work. In all branches of mathematics, he wrote more than all his writings in astronomy and in optics put together. If he wrote in optics the famous huge book, *Kitāb al-Manāẓir—The Book of Optics*, in astronomy likewise

1 Ibn al-Haytham [2], p. 4.

he wrote a huge book entitled *The Configuration of the Motions of each of the Seven Wandering Stars*.

Before coming back in some details to these contributions, let me characterize Ibn al-Haytham's research programme.

1. It is a new one, concerning the relationships between mathematics and natural phenomena, never conceived before. His aim is to mathematize every empirical science. This application of mathematics can take different forms, not only given to the different disciplines, but also in one and the same discipline.
2. It does not concern only optics, but every natural science, i.e., for the epoch, astronomy and statics.
3. Its success depends on the means—mathematical, linguistic and technical—by which mathematics control the semantic and syntactical structures of natural phenomena.

2.2 Between Ptolemy and Kepler: Ibn al-Haytham's Celestial Kinematics

To put the facts right, I will turn at first, quite briefly, to Ibn al-Haytham's astronomy. He wrote at least three books criticizing the astronomical theory of Ptolemy:

1. *The Doubts concerning Ptolemy*
2. *Corrections to the* Almagest
3. *The Resolution of Doubts concerning the* Almagest

In the *Doubts*, Ibn al-Haytham comes to the conclusion that "the configuration Ptolemy assumes for the motions of the five planets is a false one".[2] A few lines further on, he continues: "The order in which Ptolemy had placed the motions of the five planets conflicts with the theory <that he had proposed>".[3] A little later, he states: "The configurations that Ptolemy assumed for the <motions of> the five planets are false ones. He decided on them knowing they were false, because he was unable <to propose> other ones.[4]" After such comments, and many others like them in several places of his writings, Ibn al-Haytham had no option but to construct a planetary theory of his own, on a solid mathematical basis, and free from the internal contradictions found in Ptolemy's *Almagest*. For this purpose, he conceived the idea of writing his monumental and fundamental book *The Configuration of the Motions of the Seven Wandering Stars*. If we wish to characterize the irreducible inconsistencies that, according to Ibn al-Haytham, vitiate Ptolemy's astronomy, we may say that they arise from the poor fit between a mathematical theory of the planets and a cosmology; that is, the combination between mathematics and physics. Ibn al-Haytham was familiar with similar, though of course not identical, situations when, in optics, as we shall see, he encountered the inconsistency between geometrical optics and physical optics as understood not only by Euclid and Ptolemy, but also by Aristotle and the philosophers.

In *The Configuration of the Motions* he deals with the apparent motions of the planets, without ever raising the question of the physical explanation of these motions in terms of dynamics. It is not the causes of celestial motions that interest Ibn al-Haytham, but only the motions themselves observed in space and time. Thus, to proceed with the systematic mathematical treatment, and to avoid the

2 See Rashed [8], p. 13.

3 See footnote 2.

4 See footnote 2.

obstacles that Ptolemy had encountered, he first needed to break away from any kind of cosmology. Thus the purpose of Ibn al-Haytham's *Configuration of the Motions* is clear: instead of constructing, as his predecessors, a cosmology, or a kind of dynamics, he constructs the first geometrical kinematics.

A close examination of the way he organizes his exposition of planetary theory shows that Ibn al-Haytham begins by omitting physical spheres and by proposing simple—in effect, descriptive—models of the motions of each of the seven planets. As the exposition progresses, he makes the models more complicated and increasingly subordinates them to the discipline of mathematics. This growing mathematization leads him to regroup the motions of several planets under a single model. This step obviously has the effect of privileging a property that is common to several motions. In this way Ibn al-Haytham opens up the way to achieving his principal objective: to establish a system of celestial kinematics. He does so without as yet formulating the concept of instantaneous speed, but by using the concept of mean speed, represented by a ratio of arcs.

In the course of his research, which I analysed elsewhere,[5] we encounter a concept of astronomy that is new in several respects. Ibn al-Haytham sets himself the task of describing the motions of the planets exactly in accordance with the paths they draw on the celestial sphere. He is neither trying 'to save the phenomena', like Ptolemy, that is, to explain the irregularities in the assumed motion by means of artifices such as the equant; nor trying to account for the observed motions by appealing to underlying mechanisms or hidden natures. He wants to give a rigorously exact description of the observed motions in terms of mathematics. Thus his theory for the motion of the planets calls upon no more than observation and conceptual constructs susceptible of explaining the data, such as the eccentric circle and in some cases the epicycle. However, this theory does not aim to describe anything beyond observation and these concepts, and in no way is it concerned to propose a causal explanation of the motions.

The new astronomy no longer aims at constructing a model of the universe, as in the *Almagest*, but only at describing the apparent motion of each planet, a motion composed of elementary motions, and, for the inferior planets, also of an epicycle. Ibn al-Haytham considers various properties of this apparent motion: localization and the kinematic properties of the variations in speed.

In this new astronomy, as in the old one, every observed motion is circular and uniform, or composed of circular and uniform motions. To find these motions, Ibn al-Haytham uses various systems of spherical coordinates: equatorial coordinates (the required time and its proper inclination); horizon coordinates (altitude and azimuth) and ecliptic coordinates. The use of equatorial coordinates as a primary system of reference marks a break with Hellenistic astronomy. In the latter, the motion of the orbs was measured against the ecliptic, and all coordinates were ecliptic ones (latitude and longitude). Thus, basing the analysis of the planets' motion on their apparent motions drives a change in the reference system for the data; we are now dealing with right ascension and declination. Ibn al-Haytham's book thus transports us into a different system of analysis.

To sum up, in the *The Configuration of the Motions*, Ibn al-Haytham's purpose is purely kinematics; more precisely, he wanted to lay the foundations of a completely geometrical kinematics tradition. But carrying out such a project involves first of all developing some branches of geometry, as also of plane and spherical trigonometry. In both fields, Ibn al-Haytham obtained new and important results.

In astronomy, properly, there are two major processes that are jointly involved in carrying through this project: freeing celestial kinematics from cosmological

5 See Rashed [8].

connections, that is, from all considerations of dynamics, in the ancient sense of the term; and to reduce physical entities to geometrical ones. The centres of the motions are geometrical points without physical significance; the centres to which speeds are referred are also geometrical points without physical significance; even more radically, all that remains of physical time is the 'required time', that is, a geometrical magnitude. In short, in this new kinematics, we are concerned with nothing that identifies celestial bodies as physical bodies. All in all, though it is not yet that of Kepler, this new kinematics is no longer that of Ptolemy nor of any of Ibn al-Haytham's predecessors; it is *sui generis*, half way between Ptolemy and Kepler. It shares two important ideas with ancient kinematics: every celestial motion is composed of elementary uniform circular motions, and the centre of observation is the same as the centre of the Universe. On the other hand, it has in common with modern kinematics the fact that the physical centres of motions and speeds are replaced by geometrical centres.

In fact, once Ibn al-Haytham had engaged upon mathematizing astronomy and had noted not only the internal contradictions in Ptolemy, but doubtless also the difficulty of constructing a self-consistent mathematical theory of material spheres using an Aristotelian physics, he conceived the project of giving a completely geometrized kinematic account.

Ibn al-Haytham had the same experience in optics. In astronomy, kinematics and cosmology are entirely separated to effect a reform of the discipline, just as in optics, work on light and its propagation is entirely separated from work on vision to effect a reform of optics; in the one case as in the other, we shall see, Ibn al-Haytham arrived at a new idea of the science concerned.

2.3 Ibn al-Haytham's Reform of Optics

It is now time to come to Ibn al-Haytham's optics. As we have said above, Ibn al-Haytham was preceded by two centuries of translation into Arabic of the main Greek optical writings, as well of inventive research. Among his Arabic predecessors, al-Kindī, Qusṭā ibn Lūqā, Aḥmad ibn ʿĪsā ʻUṭārid, etc. During these two centuries, the interest shown in the study of burning mirrors is an essential part of the comprehension of the development of catoptrics, anaclastics and dioptrics, as the book produced between 983 and 985 by the mathematician al-ʿAlāʾ ibn Sahl testifies. Before this contribution of Ibn Sahl, the catoptricians like Diocles, Anthemius of Tralles, al-Kindī etc.[6] asked themselves about geometrical properties of mirrors and about light they reflect at a given distance. Ibn Sahl modifies the question by considering not only mirrors but also burning instruments, i.e. those which are susceptible to light not only by reflection, but also by refraction; and how in each case the focalization of light is obtained. Ibn Sahl studies then, according to the distance of the source (finite or infinite) and the type of lighting (reflection or refraction) the parabolic mirror, the ellipsoidal mirror, the plano-convex lens and the biconvex lens. In each of these, he proceeds to a mathematical study of the curve, and, then, expounds a mechanical continuous drawing of it. For the plano-convex lens, for instance, he starts by studying the hyperbola as a conic section, in order then to take up again a study of the tangent plane to the surface engendered by the rotation of the arc of hyperbola around a fixed straight line, and, finally, the curve as an anaclastic curve, and the laws of refraction.

These studies which focused on light and its physical behaviour were instrumental in the discovery by Ibn Sahl of the concept of a constant ratio, characteristic

6 See Rashed [5, 6].

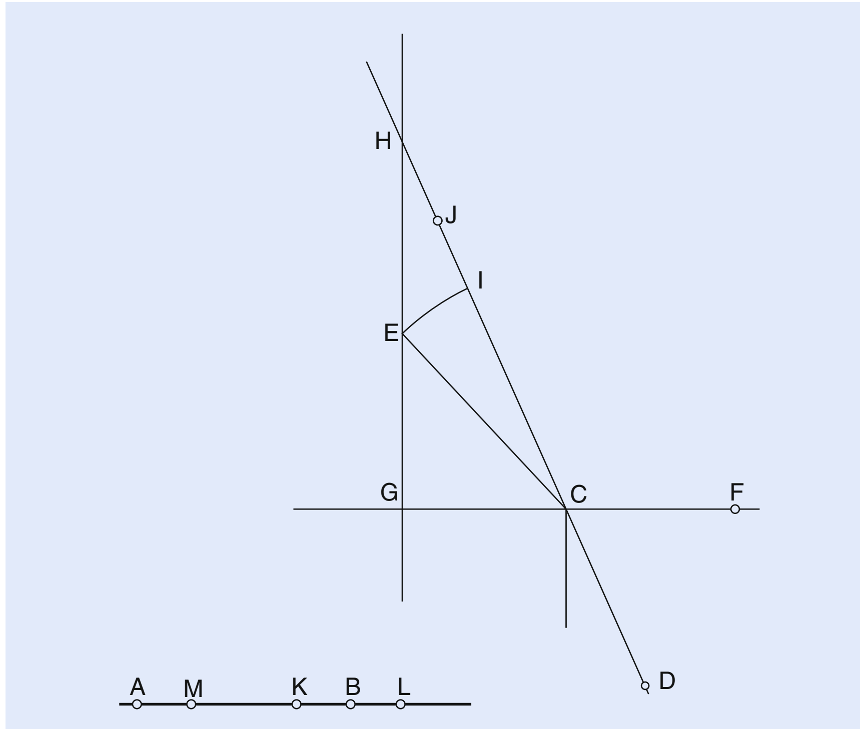

Fig. 2.1 Ibn Sahl illustration of a ray of light (*DC*) refracted as it crosses the boundary (*GF*) of two media of different refractive indices (see text for more details)

of the medium, which is a masterpiece in his study of refraction in lenses, as well as his discovery of the so-called Snellius' law.

At the beginning of his study, Ibn Sahl considers a plane surface *GF* surrounding a piece of transparent and homogeneous crystal. He next considers the straight line *CD* along which the light propagates in the crystal, the straight line *CE* along which it refracts itself in the air, and the normal at *G* on the surface *GF* which intersects the straight line *CD* at *H* and the ray refracted at *E*.

Obviously, Ibn Sahl is here applying the known law of Ptolemy according to which the ray *CD* in the crystal, the ray *CE* in the air and the normal *GE* to the plane surface of the crystal are found in the same plane (Fig. 2.1). He writes then, in a brief way, and, according to his habit, with no conceptual commentary:

> Straight line *CE* is therefore smaller than straightline *CH*. From straight line *CH*, we separate the straight line *CI* equal to straight line *CE*; we divide *HI* into two halves at point *J*; we make the ratio of straight line *AK* to straight line *AB* equal to the ratio of straight line *CI* to straight line *CJ*. We draw the line *BL* on the prolongation of straight line *AB* and we make it equal to straight line *BK*.[7]

In these few phrases, Ibn Sahl draws the conclusion first that $CE/CH < 1$, which he will use throughout his research into lenses made in the *same* crystal. In effect he does not fail to give this same ratio again, nor to reproduce this same figure, each time that he discusses refraction in this crystal (Fig. 2.2).

7 See Rashed [7], p. 106.

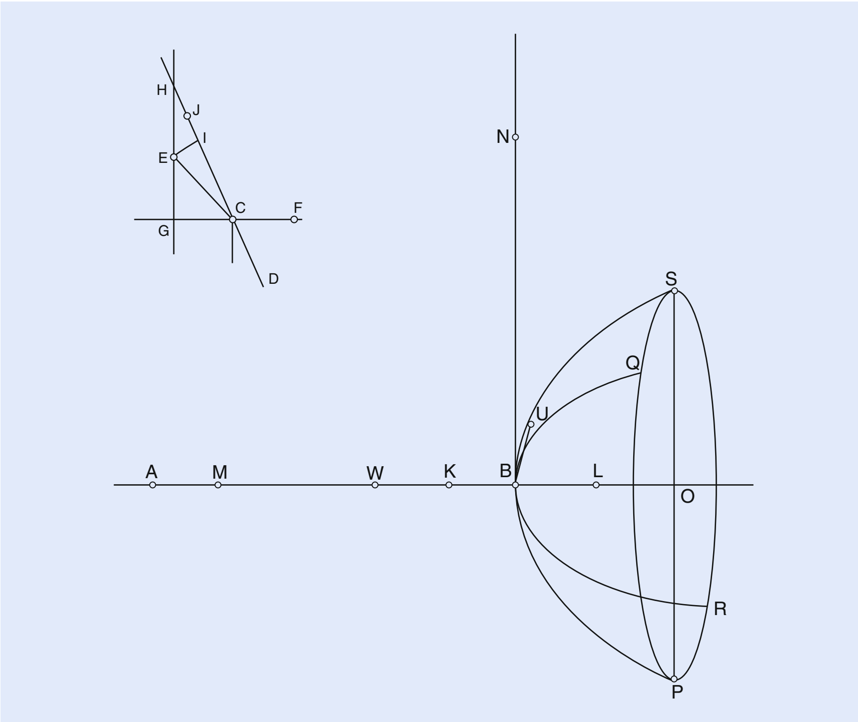

Fig. 2.2 Ibn Sahls diagram depicting refraction with plano-convex lenses (see text for more details)

But the ratio is nothing other than the inverse of the index of refraction in this crystal in relation to the air. Considering the i_1 and i_2 as the angles formed, respectively, by *CD* and by *CE* with the normal *GH*, we have

$$\frac{1}{n} = \frac{\sin i_1}{\sin i_2} = \frac{CG}{CH} \cdot \frac{CE}{CG} = \frac{CE}{CH}.$$

Ibn Sahl takes on the segment *CH* a point *I* such that *CI*=*CE*, and a point *J* at the midpoint of *IH*. This gives

$$\frac{CI}{CH} = \frac{1}{n}.$$

The division *CIJH* characterizes this crystal for all refraction.

Ibn Sahl shows, moreover, in the course of his research into the plano-convex lens and the biconvex lens, that the choice of hyperbola to fashion the lens depends on the nature of the crystal, since the eccentricity of the hyperbola is $e = 1/n$.

Thus, Ibn Sahl had conceived and put together an area of research into burning instruments and, also, anaclastics. But, obliged to think about conical figures other than the parabola and the ellipse—the hyperbola for example—as anaclastic curves, he was quite naturally led to the discovery of the law of Snellius. We understand therefore that dioptrics, when it was developed by Ibn Sahl, only dealt with matters involving the propagation of light, independently of problems of vision. The eye did not have its place within the area of burning instruments, nor did the rest of the subject of vision. It is thus an objective point of view which is deliberately adopted in the analysis of luminous phenomena. Rich in technical material, this new discipline is in fact very poor on physical content: it is evanescent and reduces a few energy considerations. By way of example, at least in his writings that have reached us, Ibn Sahl never tried to explain why certain rays change direction and are focused when they change medium: it is enough for him to know that a beam of rays parallel to the axis of a plano-convex hyperbolic lens gives by refraction a converging beam. As for the question why the focusing produces a blaze, Ibn Sahl is satisfied with a definition of the luminous ray by its action of setting ablaze by postulating, as did his successors elsewhere for much longer, that the heating is proportional to the number of rays.

Whilst Ibn Sahl was finishing his treatise on *Burning Instruments* very probably in Baghdad, Ibn al-Haytham was probably beginning his scientific career. It would not be surprising therefore if the young mathematician and physicist had been familiar with the works of the elder, if he cited them and was inspired by them. The presence of Ibn Sahl demolishes straightaway the image carved by historians of an isolated Ibn al-Haytham whose predecessors were the Alexandrians and the Byzantines: Euclid, Ptolemy and Anthemius of Tralles. Thus, thanks to this new filiation, the presence of certain themes of research in the writings of Ibn al-Haytham, not only his work on the dioptre, the burning sphere and the spherical lens, is clarified; it authorizes what was not possible previously: to assess the distance covered by a generation of optical research—a distance so much more important, from the historical and the epistemological point of view, now that we are on the eve of one of the first revolutions in optics, if not in physics. Compared with the writings of the Greek and Arab mathematicians who preceded him, the optical work by Ibn al-Haytham presents at first glance two striking features: extension and reform. It will be concluded on a more careful examination that the first trait is the material trace of the second. In fact no one before Ibn al-Haytham had embraced so many domains in his research, collecting together fairly independent traditions: mathematical, philosophical, medical. The titles of his books serve moreover to illustrate this large spectrum: *The Light of the Moon*, *The Light of the Stars*, *The Rainbow and the Halo*, *Spherical Burning Mirrors*, *Parabolical Burning Mirrors*, *The Burning Sphere*, *The Shape of the Eclipse*, *The Formation of Shadows*, *On Light*, as well as his *Book of Optics* translated into Latin in the twelfth century and studied and commented on in Arabic and Latin until the seventeenth century. Ibn al-Haytham therefore studied not only the traditional themes of optical research but also other new ones to cover finally the following areas: optics, catoptrics, dioptrics, physical optics, meteorological optics, burning mirrors, the burning sphere.

A more meticulous look reveals that, in the majority of these writings, Ibn al-Haytham pursued the realization of his programme to reform the discipline, which brought clearly to take up each different problem in turn. The founding action of this reform consisted in making clear the distinction, for the first time in the history of optics, between the conditions of propagation of light and the conditions of vision of objects. It led, on one hand, to providing physical support for the rules of propagation—it concerns a mathematically guaranteed analogy between a mechanical model of the movement of a solid ball thrown against an obstacle, and that of the light—and, on the other hand, to proceeding everywhere geometrically and by observation and experimentation. It led also to the definition of the concept of light ray and light bundle as a set of straight lines on which light propagates, rays independent from each other which propagate in a homogeneous region of space. These rays are not modified by other rays which propagate in the same region. Thanks to the concept of light bundle, Ibn al-Haytham was able to study the propagation and diffusion of light mathematically and experimentally. Optics no longer has the meaning that is assumed formerly: a geometry of perception. It includes henceforth two parts: a theory of vision, with which is also associated a physiology of the eye and a psychology of perception, and a theory of light, to which are linked geometrical optics and physical optics. Without doubt traces of the ancient optics are still detected: the survival of ancient terms, or a tendency to pose the problem in relation to the subject of vision without that being really necessary. But these relics do not have to deceive: their effect is no longer the same, nor is their meaning. The organization of his *Book of Optics* reflects already the new situation. In it are books devoted in full to propagation—the third chapter of the first book and Books IV to VII; others deal with vision and related problems. This reform led, amongst other things, to the emergence of new problems, never previously posed, such as the famous "problem of Alhazen" on catoptrics, the examination of the spherical lens and the spherical dioptre, not only as burning instruments but also as optical instruments, in dioptrics; and to experimental control as a practice of investigation as well as the norm for proofs in optics and more generally in physics.

Let us follow now the realization of his reform in the *Book of Optics* and in other treatises. This book opens with a rejection and a reformulation. Ibn al-Haytham rejects straightaway all the variants of the doctrine on the visual ray, to ally himself with philosophers who defended an intromissionist doctrine on the form of visible objects. A fundamental difference remains nevertheless between him and the philosophers, such as his contemporary Avicenna: Ibn al-Haytham did not consider the forms perceived by the eyes as "totalities" which radiate from the visible object under the effect of light, but as reducible to their elements: from every point of the visible object radiate a ray towards the eye. The latter has become without soul, without *πνεῦμα ὀπτικόν*, a simple optical instrument. The whole problem was then to explain how the eye perceives the visible object with the aid of these rays emitted from every visible point.

After a short introductory chapter, Ibn al-Haytham devotes two successive chapters—the second and the third books of his *Book of Optics*—to the foundations of the new structure. In one, he defines the conditions for the possibility of vision, while the other is about the conditions for the possibility of light and its propagation. These conditions, which Ibn al-Haytham presents in the two cases as empirical notions, i.e. as resulting from an ordered observation or a controlled experiment, are effectively constraints on the elaboration of the theory of vision, and in this way on the new style of optics. The conditions for vision detailed by Ibn al-Haytham are six: the visible object must be luminous by itself or illuminated by another; it must be opposite to the eye, i.e. one can draw a straight line to the eye from each of its points; the medium that separates it from the eye must be transparent, without being cut into by any opaque obstacle; the visible object must be more opaque than this medium; it must be of a certain volume, in relation to the visual sharpness. These are the notions, writes Ibn al-Haytham, "without which vision cannot take place". These conditions, one cannot fail to notice, do not refer, as in the ancient optics, to those of light or its propagation. Of these, the most important, established by Ibn al-Haytham, are the following: light exists independently of vision and exterior to it; it moves with great speed and not instantaneously; it loses intensity as it moves away from the source; the light from a luminous source—substantial—and that from an illuminated object—second or accidental—propagate onto bodies which surround them, penetrate transparent media, and light up opaque bodies which in turn emit light; the light propagates from every point of the luminous or illuminated object in straight lines in transparent media and in all directions; these virtual straight lines along which light propagates form with it "the rays"; these lines can be parallel or cross one another, but the light does not mix in either case; the reflected or refracted light propagates along straight lines in particular directions. As can be noted, none of these notions relate to vision. Ibn al-Haytham completes them with other notions relative to colour. According to him, the colours exist independently from the light in opaque bodies, and as a consequence only light emitted by these bodies—second or accidental light—accompanies the colours which propagate then according to the same principles and laws as the light. As we have explained elsewhere, it is this doctrine on colours which imposed on Ibn al-Haytham concessions to the philosophical tradition, obliging him to keep the language of "forms", already devoid of content when he only deals with light.[8]

A theory of vision must henceforth answer not only the six conditions of vision, but also the conditions of light and its propagation. Ibn al-Haytham devotes the rest of the first book of his *Book of Optics* and the two following books to the elaboration of this theory, where he takes up again the physiology of the eye and a psychology of perception as an integral part of this new intromissionist theory.

Three books of the *Book of Optics*—the fourth to the sixth—deal with catoptrics. This area, as ancient as the discipline itself, amply studied by Ptolemy in his *Optics*, has never been the object of so extensive a study as that by Ibn al-Haytham. Besides the three voluminous books of his *Book of Optics*, Ibn al-Haytham devotes other essays to it which complete them, on the subject of connected problems such as that of burning mirrors. Research into catoptrics by Ibn al-Haytham distinguishes itself, among other traits, by the introduction of physical ideas, both to explain the known ideas and to grasp new phenomena. It is in the course of this study that Ibn al-Haytham poses himself new questions, such as the problem that bears his name.

Let us consider some aspect of this research into catoptrics by Ibn al-Haytham. He restates the law of reflection, and explains it with the help of the mechanical model already mentioned. Then he studies this law for different mirrors: plane, spherical, cylindrical and conical. In each case, he applies himself above all to the determination of the tangent plane to the surface of the mirror at the point of incidence, in order to determine the plane perpendicular to this last plane, which includes the incident ray, the reflected ray and the normal to the point of incidence. Here as in his other studies, to prove these results experimentally, he conceives and builds an apparatus inspired by the one that Ptolemy constructed to study reflection, but more complicated and adaptable to every case. Ibn al-Haytham also studies the image of an object and its position in the different mirrors. He applies himself to a whole class of problems: the determination of the incidence of a given reflection in the different mirrors and conversely. He also poses for the different mirrors the problem which his name is associated with: given any two points in front of a mirror, how does one determine on the surface of the mirror a point such that the straight line which joins the point to one of the two given points is the incident ray, whilst the straight line that joins this point to the other given point is the reflected ray. This problem, which rapidly becomes more complicated, has been solved by Ibn al-Haytham.

Ibn al-Haytham pursues this catoptric research in other essays, some of which are later than the *Book of Optics*, such as *Spherical Burning Mirrors*.[9] It is in this essay of a particular interest that Ibn al-Haytham discovers the longitudinal spherical aberration; it is also in this text that he proves the following proposition:

On a sphere of centre E let there be a zone surrounded by two circles of axis EB; let IJ be the generator arc of this zone, and D its midpoint. Ibn al-Haytham has shown in two previous propositions that to each of the two circles is associated a point of the axis towards which the incident rays parallel to the axis reflect on this circle. He shows here that all the rays reflected on the zone meet the segment thus defined: if GD is the medium ray of the zone, the point H is associated with D, and the segment is on either side of H. The length of this segment depends on the arc IJ (Fig. 2.3).

8 See Rashed [4], pp. 271–298.

9 Ibn al-Haytham [1].

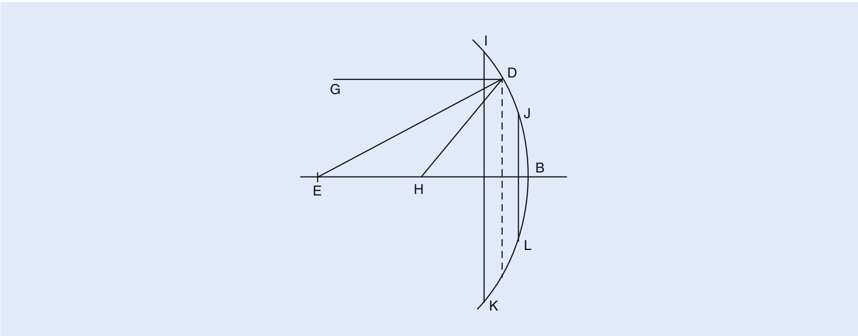

Fig. 2.3 Ibn al-Hayhtam illustration of the longitudinal spherical aberration

The seventh and last book of the *Book of Optics* by Ibn al-Haytham is devoted to dioptrics. In the same way as he did for catoptrics, Ibn al-Haytham inserts in this book the elements of a physical—mechanical—explanation of refraction. Moreover, his book is completed by his essays, such as his treatise on the *Burning Sphere* or his *Discourse on Light*, where he comes back to the notion about the medium, following Ibn Sahl.

In this seventh book of the *Book of Optics*, Ibn al-Haytham starts by taking on the two qualitative laws of refraction, and several quantitative rules, all controlled experimentally with the help of an apparatus that he conceives and builds as in the previous case. The two quantitative laws known by his predecessors, Ptolemy and Ibn Sahl, can be expressed as follows: (1) the incident ray, the normal at the point of refraction and the refracted ray are in the same plane; the refracted ray approaches (or moves away from) the normal if the light passes from a less (respectively more) refractive medium to a more (respectively less) refractive medium; (2) the principle of the inverse return.

But, instead of following the way opened by Ibn Sahl through his discovery of the law of Snellius, Ibn al-Haytham returns to the ratios of angles and establishes his quantitative rules.

1. The angles of deviation vary in direct proportion to the angles of incidence: if in medium n_1 one takes $i' > i$, one will have, in medium n_2, $d' > d$ (i is the angle of incidence, r the angle of refraction and d the angle of deviation; $d = |i - r|$).
2. If the angle of incidence increases by a certain amount, the angle of deviation increases by a smaller quantity: if $i' > i - I$ and $d' > d$, one will have $d' - d < i' - i$.
3. The angle of refraction increases in proportion to the angle of incidence: if $i' > i$, one will have $r' > r$.
4. If the light penetrates from a less refractive medium into a more refractive medium, $n_1 < n_2$, one has $d < \frac{1}{2}i$; in the opposite path, one has $d < \frac{(i+d)}{2}$, and one will have $2i > r$.
5. Ibn al-Haytham takes up again the rules stated by Ibn Sahl in his book on *The Celestial Sphere*; he affirms that, if the light penetrates from a medium n_1, with the same angle of incidence, into two different media n_2 and n_3, then the angle of deviation is different for each of these media because of the difference in opaqueness. If, for example, n_3 is more opaque than n_2, then the angle of deviation will be larger in n_3 than in n_2. Conversely, if n_1 is more opaque than n_2, and n_2 more opaque than n_3, the angle of deviation will be larger in n_3 than in n_2.

Contrary to what Ibn al-Haytham believes, these quantitative rules are not all valid in a general sense. But to his credit all are provable within the limits of the experimental conditions he effectively envisaged in his *Book of Optics*; the media are air, water and glass, with angles of incidence which do not go above 80°.[10]

Ibn al-Haytham devotes a substantial part of the seventh book to the study of the image of an object by refraction, notably if the surface of separation of the two media is either plane or spherical. It is in the course of this study that he settles on the spherical dioptre and the spherical lens, following thus in some way the research by Ibn Sahl, but modifying it considerably; this study of the dioptre and the lens appears in effect in the chapter devoted to the problem of the image, and is not separated from the problem of vision. For the dioptre, Ibn al-Haytham considers two cases, depending on whether the source—punctual and at a finite distance—is found on the concave or convex side of the spherical surface of the dioptre.

Ibn al-Haytham studies the spherical lens, giving particular attention to the image that it gives of an object. He restricts himself nevertheless to the examination of only one case, when the eye and the object are on the same diameter. Put another way, he studies the image through a spherical lens of an object placed in a particular position on the diameter passing through the eye. His procedure is not without similarities to that of Ibn Sahl when he studied the biconvex hyperbolic lens. Ibn al-Haytham considers two dioptres separately, and applies the results obtained previously. It is in the course of his study of the spherical lens that Ibn al-Haytham returns to the spherical aberration of a point at a finite distance in the case of the dioptre, in order to study the image of a segment which is a portion of the segment defined by the spherical aberration.

In his treatise on the *Burning Sphere*, Ibn al-Haytham explains and refines certain results on the spherical lens which he had already obtained in his *Book of Optics*. However, he returns to the question of the burning by means of that lens. It is in this treatise that we encounter the first deliberate study of spherical aberration for parallel rays falling on a glass sphere and undergoing two refractions. In the course of this study, Ibn al-Haytham uses numerical data given in the *Optics* by Ptolemy for the two angles of incidence 40° and 50°, and, to explain this phenomenon of focusing of light propagated along trajectories parallel to the diameter of the sphere, he returns to angular values instead of applying what is called the law of Snellius.

In this treatise on the *Burning Sphere*, as in the seventh book of his *Book of Optics* or in other writings on dioptrics, Ibn al-Haytham exposes his research in a somewhat paradoxical way: while he takes a lot of care to invent, fashion and describe some experimental devices that are advanced for this age, allowing the determination of numerical values, in most cases he avoids giving these values. When he does give them, as in the treatise on the *Burning Sphere*, it is with economy and circumspection. For this attitude, already noted, at least two reasons can perhaps be found. The first is in the style of the scientific practice itself: quantitative description does not yet seem to be a compelling norm. The second is no doubt linked: the experimental devices can only give approximate values. It is for this reason that Ibn al-Haytham took into account the values which he had borrowed from the *Optics* by Ptolemy.

This book on the *Burning Sphere* is undoubtedly one of the summits of research in classical optics. Kamāl al-Dīn al-Fārisī (d. 1319) was able to put this book to work in order to explain for the first time the rainbow and the hallo. In this book, Ibn al-Haytham returns to the problem of combustion with the help of a spherical lens. Here then, is a text that enables us to follow the evolution of Ibn al-Haytham's thought on spherical lenses, by examining how he takes up the problem raised by his predecessor Ibn Sahl: to cause combustion by refraction,

10 See Rashed [3].

with the help of a lens. For Ibn al-Haytham, this research is an integral part of optics.

He begins this book by proving several propositions two of which are particularly important:

1. $\frac{i}{4} < d < \frac{i}{2}$ (i, angle of incidence in the glass; d, angle of deviation)
2. Let α and β be two arcs of a circle; we suppose that $\alpha > \beta$; $\alpha = \alpha_1 + \alpha_2$ and $\beta = \beta_1 + \beta_2$, such that

$$\frac{\alpha_2}{\alpha_1} = \frac{\beta_2}{\beta_1} = k < 1$$

and

$$\alpha_1 < \frac{\pi}{2}$$

Then $\dfrac{\sin\beta_1}{\sin\beta_2} > \dfrac{\sin\alpha_1}{\sin\alpha_2}$.

With the help of these two propositions, as well as his rules of refraction, Ibn al-Haytham studies the propagation of a bundle of parallel rays falling upon a glass or crystal sphere. Let us sketch how he proceeds.

In a first proposition, he shows that all parallel rays falling on the sphere with one and the same angle of incidence converge, after two refractions, towards one and the same point of the diameter which is parallel to the ray. This point is the focus associated with incidence i.

Thus, he considers a ray (HN) parallel to the diameter AC, falling upon the sphere at M. The refracted ray corresponding to it meets the sphere at B, and meets AC at point S. Point S is the focus associated with incidence i, and it belongs to the segment $[CK]$ where K is the intersection of MB with AC (Fig. 2.4).

In a second proposition, he proves that the total deviation is twice each of the deviations: $D = 2d$.

He proves then that a given point S, beyond C on the diameter, can be obtained only from a single point M; that is to say, S corresponds to a single incidence.

In a third proposition, he proves that the two incidences i and i', correspond two distinct points S and S'.

In a fourth proposition, he proves: for $i > i'$, we have S and S' such that $CS' > CS$. Therefore, when i increases, CS decreases. To a given point S, therefore, there corresponds one single incidence i.

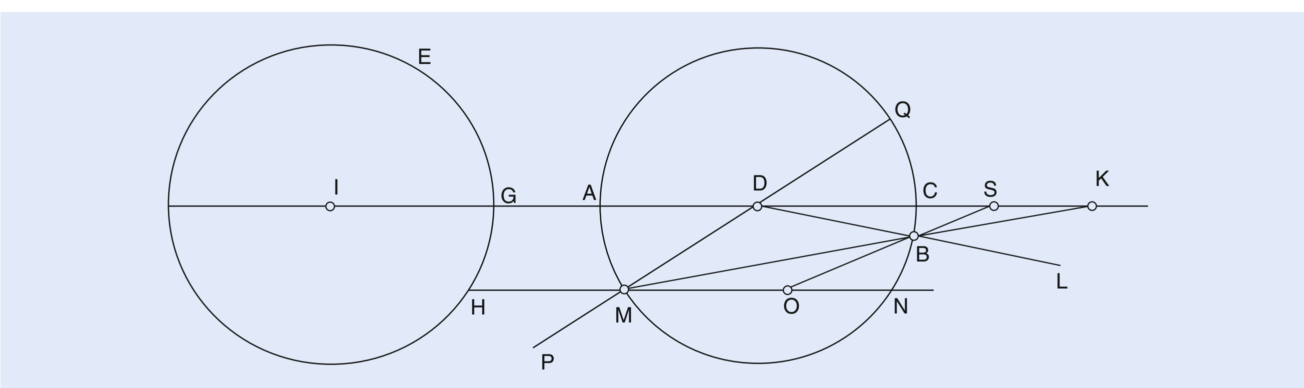

Fig. 2.4 Illustration of spherical aberration in glass (crystal) spheres

Ibn al-Haytham proposes to determine the extremities of the segment on which the points *S* are located. With this in view, he studies the positions of *B*—the point of the second refraction—when the angle of incidence varies. As far as we know, this is the first deliberate study of spherical aberration for parallel rays which fall on a glass sphere and undergo two refractions.[11]

2.4 Conclusion

Let us stop at this point on spherical aberration, to conclude.

With Ibn al-Haytham, one result has been definitively obtained: the half century which separates him from Ibn Sahl should be counted among the distinctive moments in the history of optics: dioptrics appears to have extended its domain of validity and, by its very progress, to have changed its orientation. With Ibn al-Haytham, the conception of dioptrics as a geometry of lenses has become outdated. Here again, in his own words, we must combine mathematics and physics in order to study dioptres and lenses, whether burning or not. The mathematization could only be achieved with Ibn al-Haytham because he separated the study of the natural phenomenon of light from vision and sight. The step taken suggests already that the domain carved out by Ibn Sahl was not long-lived and wound up, 50 years later, exploding under the assault of the mathematician and physicist Ibn al-Haytham. In optics as in astronomy the research programme of Ibn al-Haytham is the same: mathematize the discipline and combine this mathematization with the ideas of the natural phenomena.

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11 See Rashed [7], p. 164.

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